PUBLICATIONS OF THE
ASTRONOMICAL SOCIETY OF THE PACIFIC

Vol. 86 August 1974 No. 512

DENSITY-WAVE THEORY OF THE SPIRAL STRUCTURE OF GALAXIES*

ROLAND WIELEN
Astronomisches Rechen-Institut, Heidelberg, Germany

Received 10 April 1974

This article reviews the gravitational interpretation of the spiral structure of galaxies in terms of density waves. After stating the basic problems to be explained, we describe the theory of density waves in stellar and gaseous disks of galaxies, and discuss the merits and difficulties provided by the concept of density waves for our present understanding of spiral galaxies.

Key words: stellar dynamics — galaxies — spiral structure — density waves

I. Introduction

Spiral structure is the most striking morphological property of galaxies. Sixty-one percent of the galaxies apparently brighter than $m_{pg} = 12.7$ show spiral structure (de Vaucouleurs 1963). The various forms of spiral structure are used to define the basic classification scheme for galaxies. The type of spiral structure turns out to be strongly correlated with other physical properties of a galaxy, such as the relative gas content and the frequency of young objects. Hence the spiral structure is at least a very sensitive indicator for many basic properties of these stellar systems. Therefore, the theoretical interpretation of spiral structure represents an important and necessary step towards the physical understanding of the entire structure of a galaxy.

This paper will review the progress that has been made during the last decade toward a gravitational theory of spiral structure. The spiral arms will be viewed as density waves which are created and maintained by gravitational forces. This interpretation of spiral structure as a density-wave pattern was first developed by B. Lindblad some decades ago. In recent years, however, the density-wave theory has received a completely new impulse, especially through the eminent work of C. C. Lin and his collaborators. The density-wave theory is now able to give a rather successful interpretation of the observed spiral structure of galaxies. Of course, that does not necessarily mean that all spiral arms in galaxies must be density waves. In fact, there are indications that other mechanisms — such as tidal interactions of galaxies, or explosions in the nuclei of galaxies, or magnetic confinement — are (or at least may be) responsible for some of the observed spiral arms of galaxies. These mechanisms, however, which produce essentially material arms, seem to be important only under exceptional circumstances which are not representative for the majority of the disk galaxies. Density waves are probably the most general phenomenon producing spiral structure. Furthermore, the density-wave concept is presently the only one which is able to explain a long duration of spiral structure, while all the other mechanisms produce rather transient spiral phenomena.

We will restrict ourselves to normal spirals (Hubble type S, de Vaucouleurs type SA) and not consider barred spirals (SB). The arms of normal spirals are conspicuous because of the young stars, the gas, and the dust which they contain. Otherwise the arms are only minor perturbations in the smooth, rotationally sym-

*One in a series of invited review articles currently appearing in these *Publications.*

metric mass distribution of S galaxies. By contrast, the bar in an SB galaxy seems to represent a larger deviation from rotational symmetry, since even the old stellar population is strongly involved in the bar and because large deviations from circular motion are observed in SB galaxies. The bar of an extreme SB galaxy may be a material arm, not a density wave. There may be, however, a gradual transition from S- to SB-galaxies (de Vaucouleurs 1959; Lin 1971). A survey of the problems connected with the interpretation of barred spirals is given by E. M. Burbidge (1970), Freeman (1970), and de Vaucouleurs and Freeman (1972).

II. Statement of Problems

Which are the main problems we encounter in the interpretation of the spiral structure of galaxies?

A. *Grand Design.* Spiral arms appear to be locally often very irregular and patchy. On a large scale, however, the regularity and symmetry of spiral structure is usually striking. This 'grand design' of spiral structure is the primary feature to be explained. The grand design rules out a purely local theory and demands a global theory, in which a galaxy as a whole must be considered. The spiral arms of galaxies can take various forms, going from rings to tightly-wound spirals to wide-open patterns. Usually there are two symmetric main arms. In the outer regions of a galaxy, however, multiple-armed structures often coexist with the basic two-arm pattern. The large variety of forms of spiral structure can be explained by a unified theory only if that theory is rather flexible.

B. *Permanence.* It is now generally assumed that the type of spiral structure in a galaxy does not vary with time to any remarkable degree. This is indicated by the correlation between spiral type and basic properties of a galaxy such as total mass or relative gas content, which cannot change rapidly. On the other hand, most galaxies rotate differentially: the angular velocity of rotation, $\Omega(R)$, varies strongly with the distance R from the center of the galaxy. If a spiral arm consisted always of the same stars and gas clouds, then within a few rotation periods it would be wound up. This winding dilemma can be resolved by the concept of density waves. A density wave can move relative to the stars and gas, and does not have to follow the differential rotation of the galactic material. Although the density-wave concept overcomes the winding dilemma, it does not automatically guarantee the persistence of spiral structure, since the wave can be damped by various effects.

C. *Origin.* Spiral structure cannot have been present in the protogalaxy, because at the beginning a galaxy was probably turbulent and not very flattened. Hence the spiral structure must have originated after the end of the early phase of galactic evolution. Because of the various damping effects, even a once-existing density wave will need an energetic replenishment, either continuously or at least from time to time.

D. *Differentiation of Stars and Gas.* Spiral structure is shown in a striking way only by the gas and young objects; older stars do, at most, mildly participate in the spiral pattern. This is a particular problem for the gravitational density-wave concept, because we must show why the same gravitational field produces different effects for stars and gas. Furthermore, the youngest stars and the H II regions usually form a narrow line within the broader H I arms, much smaller than the spacing of the arms. Hence the process of star formation seems to work rather selectively within the gas.

III. Inefficiency of Magnetic Fields

For a long time, most astronomers believed in a magnetohydrodynamic interpretation of spiral arms, although there was never a well-developed and fully convincing theory of that kind available. Nowadays, the weakness of the measured magnetic fields rules out that the magnetic field is responsible for the grand design of spiral structure in our Galaxy. From the measurements of Faraday rotation in pulsars and other radio sources, and from the Zeeman splitting of the 21-cm line of H I, a magnetic field strength of the order of a few microgauss, typically $B \sim 3\ \mu G$, is found (Woltjer 1965, 1970; G. R. Burbidge 1969). The energy density of the magnetic field, $B^2/8\pi$, is then at most comparable to the energy density of the random ('turbulent') motions of the gas clouds, $\rho_{gas}v^2_{turb}/2$, while the energy density of the galactic rotation of the gas, $\rho_{gas}v^2_{rot}/2$, is at least two orders of magnitude larger than $B^2/8\pi$. Hence the magnetic field is too weak for causing any extremely large deviation from circular motion, as would be required, e.g., for rigidly rotating material arms. Even a much stronger field (e.g., 30 μG) would probably not be rigid enough to stop the winding-up of material arms. Locally, however, the magnetic forces are certainly of importance for the dynamics of the gas. Unfortunately, magnetic fields seem to hamper gravitational density waves in the gas rather than to assist them (Roberts and Yuan 1970). For external S galaxies, we have only very vague estimates on the strength of magnetic fields in the disks (e.g., Mathewson, van der Kruit, and Brown 1972), but our local magnetic field does not seem to be exceptionally weak or strong.

IV. Basic Concept of Density Waves

The winding dilemma can be avoided by assuming that the spiral arms do not continually consist of the same gas clouds and stars, but that the spiral arms at any moment of time represent the local maxima of a density wave in the galaxy. This density wave includes both the gas and the stars and moves relative to these objects. The density wave is caused and maintained by purely gravitational effects, not by pressure variations. Hence the density waves in galaxies are completely different from ordinary sound waves which are pressure-supported. The propagation of a once existing density wave is mainly due to the corresponding wave perturbation in the galactic gravitational field. An analogy to the density waves in galaxies is shown by waves in collisionless plasmas, which are created and propagated there by electromagnetic fields.

It was the late Bertil Lindblad who carried out the pioneering studies on the gravitational interpretation of the spiral structure of galaxies, during the years from about 1925 to 1964. After 1940, he developed the concept of density waves for explaining spiral arms. A convenient summary of his conclusions may be found in his review on galactic dynamics (B. Lindblad 1959). Lindblad and his co-workers concentrated on the investigation of the galactic orbits of individual stars and on resonance effects associated with these orbits. Lindblad did not succeed, however, in establishing a quantitative treatment of collective effects in a galaxy. Lin and his associates, in a series of papers starting in 1964, were the first to produce quantitative results on density waves as a collective phenomenon (Lin 1966, 1967*a,b,* 1968, 1970*a,b,* 1971; Lin and Shu 1964, 1966, 1967, 1971; Lin, Yuan, and Shu 1969). Lin's theory of density waves profited by mathematical methods that had been developed and successfully used for analogous problems in plasma physics and hydrodynamics.

Lin starts with the hypothesis of a quasi-stationary spiral structure (a term introduced by B. Lindblad (1963)), according to which the form of the spiral structure of a galaxy does not vary with time. Lin postulates that this quasi-stationary spiral structure is due to a neutral, rigidly rotating density wave. The assumption that the density wave rotates rigidly with a constant angular frequency Ω_p excludes any variation with time of the geometrical form of the spiral, such as winding up of the wave. The assumption that the wave is neutral, i.e., that its amplitude neither increases nor decreases with time but remains constant, allows an optimal permanence of the spiral pattern and preserves the contrast between the spiral arms and the underlying galactic disk. The principal aim of Lin's theory is to show that a neutral, rigidly rotating density wave can exist in a differentially rotating galaxy and that many observed properties of spiral galaxies can be accounted for by the presence of such a density wave.

For simple kinematical reasons, based on the conservation of mass and mathematically described by the equation of continuity, a density wave in a galaxy must be necessarily connected with systematic deviations of the mean velocity of stars and gas from purely circular rotation. The material must stream in a cyclic way: first it streams toward the density maximum and stays in the region of high density (spiral arm) relatively longer than in low-density regions; it then leaves the density maximum, crosses the region of low density in a relatively shorter time, and starts the cycle again. For a neutral and rigidly rotating density wave, the streamlines of the mean motion of the stars and gas are expected not to be circles any more, but still nearly closed curves (distorted circles) in a frame which corotates with the density-wave pattern. The orbits of individual stars and gas clouds are, of course, more complicated in general. It should be emphasized that a density wave by its very nature does affect the distribution of matter not only in the positional space but also in the velocity space.

In the following sections, we shall outline the present theory of galactic density waves and some of its applications in a descriptive way, avoiding a mathematical treatment of the subject. Excellent reviews of the density-wave theory with emphasis on the mathematical aspects of the theory have been given especially by Contopoulos (1970*a*, 1972, 1973*a*).

V. Undisturbed Galaxy

A density wave may be considered as a perturbation in the spatial distribution and in the motion of the material of an otherwise rotationally symmetric and stationary galaxy. Before we can analyze such a perturbation, the undisturbed state of a galaxy must be known. We restrict ourselves to ideal flat galaxies. The finite thickness of a spiral galaxy in the z-direction and its effect on density waves is usually considered as a small correction (Lin et al. 1969; Lin 1970*b*; Lin and Shu 1971; Vandervoort 1970*b*).

The observed mean rotational velocity $v_{rot}(R)$ of a galaxy is usually identified with the circular velocity, $v_{rot} \sim v_{circ} = \Omega(R)R$. From the run of $v_{circ}(R)$, a mass model of the galaxy can be derived (e.g., Toomre 1963; Schmidt 1965; Shu, Stachnik, and Yost 1971; Nordsieck 1973*a,b*). Such a mass model gives, e.g., the surface density $\sigma(R)$ as a function of R and the axisymmetric gravitational field of the galaxy. From observations, nothing reliable is known about the velocity distribution of stars in spiral galaxies except for the solar vicinity. Hence we have much freedom in constructing theoretically a self-consistent dynamical model of a galaxy. In such a dynamical model, the velocity distribution should be everywhere consistent with the assumed stationary state of the galaxy (Fricke 1952; Perek 1966; Ng 1967; Shu 1969; Vandervoort 1970*a,c*; Miyamoto 1974; and others). As a first approximation, an ellipsoidal Schwarzschild velocity distribution (see e.g., Oort 1965) is often adopted. In determining the variation of the velocity dispersion with R, Lin and his collaborators usually assume that the velocity dispersion is everywhere just equal to the critical velocity dispersion according to Toomre's stability criterion (discussed in section VII). This case is most favorable for density waves. The uncertainty about the unperturbed state of a galaxy affects, of course, the reliability of statements about the behavior of density waves.

In the zeroth approximation, the orbits of stars and gas clouds in galaxies are strictly circular. Small deviations from circular orbits can be described in a first approximation by the epicyclic theory (see e.g., B. Lindblad 1959; Oort 1965): a star moves on a small elliptic epicycle whose center rotates on a circle around the galactic center (Fig. 1). Mathematically, this means that the star makes harmonic oscillations around the mean circular-orbit position. The frequency $\kappa(R)$ of this oscillation differs in general from the circular rotation frequency $\Omega(R)$. The 'epicyclic frequency' $\kappa(R)$ can be derived, however, from the observable run of $\Omega(R)$ according to

$$\kappa = 2\Omega(1 + 1/2\, R/\Omega\, d\Omega/dR)^{1/2} \quad . \qquad (1)$$

The axial ratio of the epicycle is equal to $\kappa/(2\Omega)$, and the diameter of the epicycle is proportional to the peculiar velocity of the star.

The rotational frequency $\Omega(R)$ and the epicyclic frequency $\kappa(R)$ represent locally the natural frequencies for free oscillations of stars and gas clouds. At certain locations in a spiral galaxy, there can occur resonances between these basic frequencies Ω and κ of the unperturbed

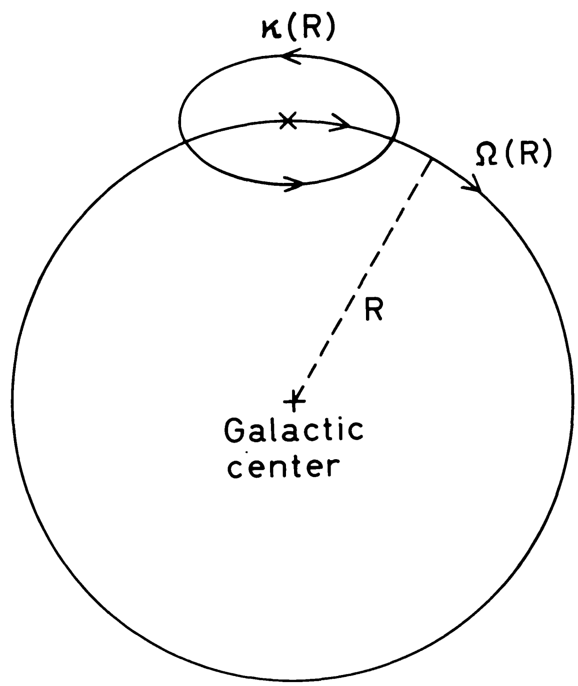

FIG. 1 — Epicycle of a stellar orbit.

galaxy and the rotational frequency Ω_p of the density-wave pattern. In general, resonances will occur if $(\Omega - \Omega_p)/\kappa$ is rational. The most important resonances are the following:

(a) corotation: $\Omega_p = \Omega$, (2)

(b) inner Lindblad resonance: $\Omega_p = \Omega - \frac{\kappa}{2}$, (3)

(c) outer Lindblad resonance: $\Omega_p = \Omega + \frac{\kappa}{2}$. (4)

For a given run of $\Omega(R)$ and $\kappa(R)$, the radius R where a resonance actually occurs, depends on the value of Ω_p. In Figure 2, the functions $\Omega(R)$, $\kappa(R)$, and $\Omega - (\kappa/2)$, $\Omega + (\kappa/2)$ are shown for the mass model of our Galaxy constructed by Schmidt (1965).

In order to illustrate the peculiarity of the resonances, we shall study the stellar orbits in a frame of reference which rotates with the angular frequency Ω_p relative to an inertial system (Fig. 3). In this rotating Ω_p system, the spiral pattern of the density wave is at rest. The orbit of a star away from resonances is a nonperiodic rosette. In the case of resonances, the orbit degenerates into a periodic orbit, fixed in the Ω_p system. At the Lindblad resonances, the orbits of stars are ovals, centered on the galactic center. At corotation, only the epicyclic ellipse remains. For the periodic orbits occurring at the resonances, the perturbation of an orbit by the gravitational field of the density wave will be exactly the same after each revolution. Hence the orbits will be changed secularly. In Figure 3, we have schematically indicated the unperturbed orbits. The orbits at the resonances including the perturbing effect of a density wave have been investigated by Barbanis (1970), Contopoulos (1970*b*, 1973*b*), Lynden-Bell (1973), and Vandervoort (1973).

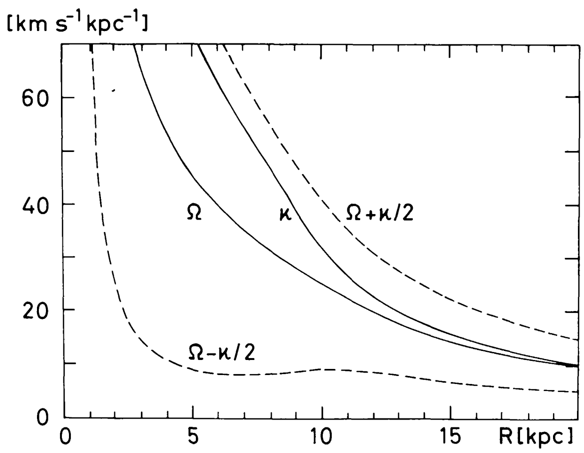

FIG. 2 — Rotational frequency Ω and epicyclic frequency κ in our Galaxy.

VI. Kinematical Density Waves

Before we embark on the study of dynamical density waves, where the gravitational field of the wave itself is taken into account, we shall first describe the simple, educative, but not very realistic case of kinematical density waves, where the self-gravity of the wave is completely neglected. The essential properties of kinematical density waves have been pointed out by B. Lindblad (1959). Our presentation will closely follow an illustrative paper by Kalnajs (1973*b*).

B. Lindblad pointed out that the quantity $\Omega - (\kappa/2)$ is often rather constant over a large range of R, say from 4 kpc to 15 kpc for our Galaxy (Fig. 2). If we choose the value of Ω_p close to that privileged number ($\sim$8 km sec^{-1} kpc^{-1}), then we have a very broad resonance region, instead of merely a circle, for the inner Lindblad resonance. From section V we know that the orbits of stars in the rotating Ω_p system are *fixed*

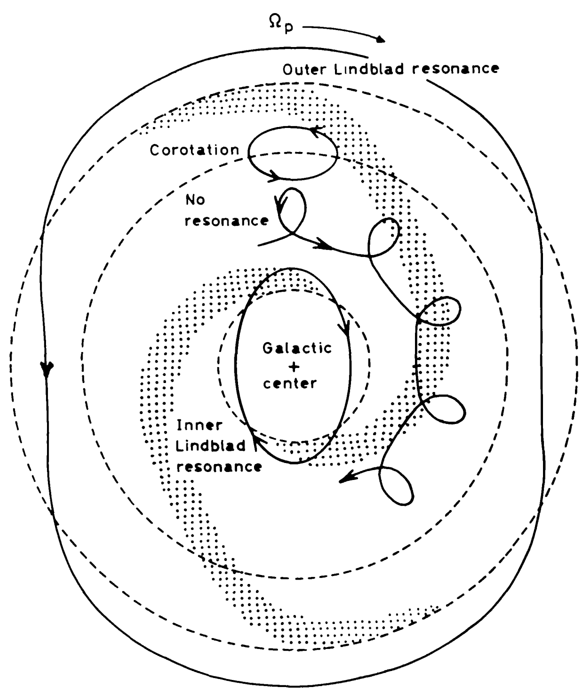

FIG. 3 — Resonant orbits in the rotating Ω_p system.

ovals (approximately ellipses) if $\Omega_p = \Omega - (\kappa/2)$. Let us suppose that $\Omega - (\kappa/2)$ were strictly constant and equal to Ω_p. Consider a sample of slightly eccentric orbits with parallel major axes, such as shown in Figure 4a. We fill these orbits uniformly with stars. Because of the law of area, the density of stars will vary along an orbit, but this effect is small for nearly round orbits and can be neglected here. The stars in different orbits revolve along the ellipses with quite different periods, namely $4\pi/\kappa$. Nevertheless, we have created a *stationary* elliptical distortion of the galaxy: Viewed from an inertial system, all the orbits and hence also the overall distortion, will precess with the same rate $\Omega_p = \Omega - (\kappa/2)$. Let us now make kinematical density waves of spiral appearance. For that purpose, we rotate the ellipses shown in Figure 4a by an angle $\theta(R)$, where θ increases rapidly but monotonically with the mean radius R of an orbit. The result is shown in Figure 4b. The density at a given point is approximately proportional to the distance between two neighboring orbits. We find two bisymmetric spiral arms of rather high density. The spiral pattern is stationary: With respect to an inertial system, the density wave rotates rigidly with an angular frequency $\Omega_p = \Omega - (\kappa/2)$, since all the individual elliptic orbits precess with this frequency. The wave character of the spiral pattern is obvious from the fact that each star rotates around the center with the frequency $\Omega(R)$, which is much larger than Ω_p.

Although the precessional motion of the ellipses is always in the direction of the general rotation of the galaxy, we can make either trailing or leading spirals by the outlined procedure. Let us consider the case of a trailing spiral. Then the sense of rotation in Figure 4b is clockwise. A star enters a spiral arm with $\dot{R} < 0$ and reaches the maximum density approximately at that point where it crosses the mean radius of its orbit while moving inward. Hence the systematic motion of the stars in a spiral arm is radially inward. The largest outward motion is reached in between the two arms, where the star crosses its mean orbital radius while traveling outward. At its perigalacticum, the tangential motion of a star is larger than the corresponding circular velocity, and Figure 4b indicates that this point lies slightly outside of an arm. The apogalacticum, with smaller tangential velocity, occurs at the inside of the arm. All these effects are of purely kinematical nature, since we have neglected the gravity of the spiral arms.

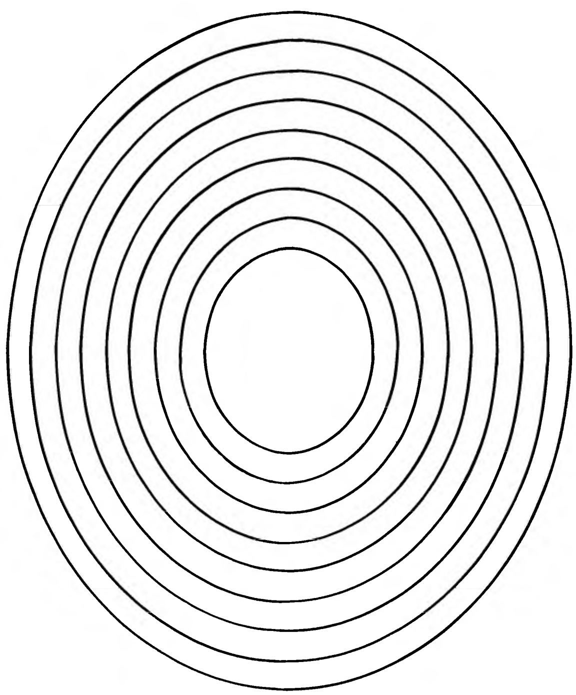

FIG. 4a — Dispersion orbits.

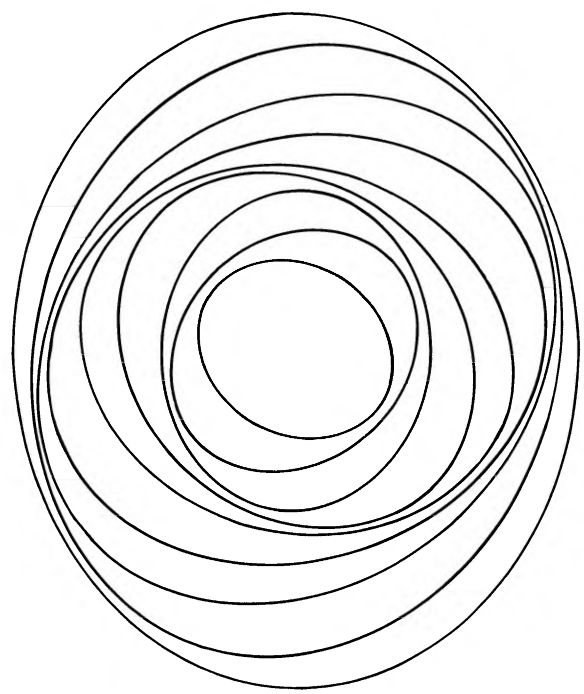

FIG. 4b — Kinematical density waves (Kalnajs).

In the dynamical theory of density waves, in which the gravity of the spiral arms is taken into account, the pattern speed Ω_p may differ significantly from the value of $\Omega - (\kappa/2)$, because the gravitational field of the wave itself can stabilize the wave even for 'nonresonant' frequencies Ω_p. In fact, Lin's theory does not need such a global resonance as is necessary for Lindblad's kinematical waves.

VII. Instabilities and Waves

Let us now investigate the general dynamical problem of density waves and of gravitational instabilities in galaxies. We consider a (small) perturbation in the structure of an otherwise rotationally symmetric and stationary galaxy. How does this perturbation behave in space and time in the future? We consider any quantity q, e.g., the density or the mean velocity, as a function of position and time, and write

$$q(\text{perturbed}) = q_0(\text{unperturbed}) + q_1(\text{perturbation}) \quad . \qquad (5)$$

As the first (linear) approximation, we assume that $|q_1| \ll |q_0|$. Then we can linearize the relevant differential equations that describe the behavior of the galactic material (for example, the Liouville equation for the stars). These linearized equations are, as usual, easier to solve than the general nonlinear problem. The time dependence of a solution for q_1, at a fixed position in an inertial system, can be described in the linear case by an exponential function

$$q_1(t) \propto \mathrm{Re}\{e^{i\omega t}\} \quad . \qquad (6)$$

The frequency $\omega = \omega_R + i\omega_I$ will be complex in general. Two main types of perturbations show up, according to the ratio of ω_R to ω_I, namely oscillations ('modes', $|\omega_R| \gg |\omega_I|$) and instabilities ($-\omega_I \gg |\omega_R|$). The amplitude of an oscillatory mode can be damped ($\omega_I > 0$), neutral ($\omega_I = 0$, i.e., ω real) or growing ('overstability', $\omega_I < 0$). The most important instability is a local gravitational collapse (Jeans instability). The general solution for q_1 is a linear superposition of all possible types of perturbation.

For explaining spiral structure, we are interested in wave solutions (modes). On the other hand, it must be ensured that the galaxy is stable against a local gravitational collapse, because otherwise a density wave would start a local collapse, which in turn would destroy the wave. Toomre (1964) has shown that a galaxy is locally stable against gravitational collapse if the local dispersion of the peculiar stellar velocities in the radial direction is greater than the critical value

$$\langle v_{\text{rad}}^2 \rangle_{\text{crit}}^{1/2} = 3.36\, G\sigma/\kappa \qquad (7)$$

Here, σ is the surface density, κ the epicyclic frequency, and G the gravitational constant.

In the solar neighborhood, equation (7) leads to 52 km sec^{-1} for the critical velocity dispersion. By taking the finite thickness of our Galaxy into account (Lin et al. 1969; Vandervoort 1970*c*; Toomre 1974), the critical value is lowered to 37 km sec^{-1} or even less. The observed velocity dispersion in the radial direction for a representative mixture of nearby stars is about 39 km sec^{-1}. However, these stars are sampled close to the galactic plane ($z \sim 0$). Due to the observed positive correlation between the z motions, perpendicular to the plane, and the motions parallel to the plane, we obtain about 48 km sec^{-1} for the z-integrated value of the observed root-mean-square radial speed of disk stars (Wielen 1974). Hence the ratio Q between the actual and critical velocity dispersion seems to be slightly larger than one ($Q \sim 48/37 \sim 1.3$). We conclude that our Galaxy is locally stable ($Q \gtrsim 1$). Also from theoretical arguments, we would expect $Q \gtrsim 1$: If the velocity dispersion in a galaxy starts out below the critical value, then the resulting Jeans instability leads to some kind of fast relaxation and increases thereby the stellar velocities. When the critical value is reached ($Q \sim 1$), then the dispersion does not increase any more, for lack of a Jeans instability. However, global instabilities or other accelerating processes may increase the velocity dispersion beyond its local critical value. The interstellar gas maintains its small velocity dispersion because of dissipation (e.g. collisions of clouds) and the stabilizing effect of the stars, which provide the major part of the total gravitational field. In the following, we shall follow Lin and his coworkers in discussing density waves mainly for those galaxies which are marginally stable ($Q = 1$), although this might be a slight oversimplification of the real situation.

VIII. Numerical Experiments

The most direct way of investigating all types of density waves and instabilities in galaxies is the detailed numerical simulation of the dynamical evolution of galaxies. In such computer experiments, the motion of the galactic matter (stars and gas) in its own gravitational field is followed stepwise by a numerical integration scheme (P. O. Lindblad 1960, 1962; Miller and Prendergast 1968; Miller, Prendergast, and Quirk 1970; Miller 1971; Quirk 1971; Hohl and Hockney 1969; Hohl 1970, 1971*a,b*, 1972*a,b*, 1973). The results obtained so far confirm qualitatively the existence of spiral density waves. Up to now, however, the numerical experiments have not yielded reliable quantitative results of the desired nature. Most of the experimental galaxies end up with a rather high velocity dispersion. This unrealistic behavior of the disk stars in the experiments is closely related to the unsolved question of how a galactic disk is stabilized against global and nonaxisymmetric disturbances. Many of the experimental galaxies do form a central oval or bar-shaped core. In agreement with analytic studies by Kalnajs (1972*a*), the experiments show clearly that the critical velocity dispersion ($Q \sim 1$) is by no means sufficient to stabilize a galactic disk of stars against all types of instabilities (Toomre 1974). It has been suggested that a massive halo of high-velocity stars may be necessary for stabilizing the galactic disk of low-velocity stars (Ostriker and Peebles 1973).

Roughly speaking, numerical experiments are much more sensitive than analytic investigations against an undue assumption on the basic structure of a galaxy. In numerical experiments, we get immediately involved in the general solution of the posed problem, i.e., the full spectrum of instabilities and waves shows up. In an analytic treatment, we can single out certain modes which we believe to be important and realistic; and we may simply ignore unwarranted components of the general solution, which are admittedly allowed by a simplified mathematical model of a galaxy, but which are not observed in a *real* galaxy. In any case, the presently available quantitative results on density waves are essentially based on analytic studies, not on numerical experiments. The contribution of numerical experiments to our understanding of density waves may grow, however, in the future.

IX. Lin's Mode

Lin and his coworkers consider, among the many possible kinds of density waves, a special mode in the form of a rigidly rotating, neutral, tightly wound spiral. The perturbation V_1 that this density wave causes in the gravitational potential V of a galaxy, is assumed to be

$$V_1(R, \theta, t) = \hat{V}_1(R) e^{i(\omega t - m\theta + \Phi(R))} \quad . \qquad (8)$$

Here, R and θ are galactocentric polar coordinates of the point under consideration (θ measured in a nonrotating, inertial system); $\hat{V}_1(R)$ is the amplitude of the wave in the potential and is supposed to vary only slowly with R; ω (real and constant) is the frequency of oscillation of V_1 at any fixed position in the galaxy; m is the number of spiral arms (usually $m = 2$); and $\Phi(R)$ is the radial phase of the wave at the distance R. Since in one rotation of the spiral pattern, m arms (i.e., minima of V_1) pass by a fixed position, the spiral pattern rotates with the frequency

$$\Omega_p = \omega/m \quad . \qquad (9)$$

At a given time and on a circle $R =$ constant, the potential V_1 has the form of an m-fold sinusoidal wave

$$V_1 = \hat{V}_1 \cos\{m[\theta - \theta_0(R, t)]\} \quad . \qquad (10)$$

The geometrical form of the spiral field can be represented by the local minima of V_1. At a distance R, the minima of V_1 occur for $m = 2$ at

$$\theta_p(R) = \tfrac{1}{2}\,\Phi(R) + \Omega_p t - 90° \qquad (11)$$

and at $\theta_p + 180°$. The form of the spiral is thus completely determined by the phase function $\Phi(R)$. If Φ were independent of R, a radial structure similar to a wagon spoke would result. To make a tightly wound spiral, $\Phi(R)$ must vary rapidly and monotonically with R. The function $\Phi(R)$ also determines the local angle ψ between the spiral arm and the circle $R =$ constant

$$\tan\psi(R) = m/(R\, d\Phi/dR) \quad . \qquad (12)$$

Furthermore, we can derive from $\Phi(R)$ the local radial wavelength λ of the wave

$$\lambda = |2\pi/(d\Phi/dR)| \quad . \qquad (13)$$

The wavelength λ coincides approximately with the radial spacing between neighboring spiral arms.

In Figure 5, the scheme for calculating the properties of the density wave is indicated. Lin adopts the perturbation V_1 of the potential according to equation (8). Then he considers first the 'response problem': What density wave is created among the stars and the gas by the assumed wave V_1 in the potential? This problem can be handled for the stars by means of the encounterless Liouville equation (see e.g., Contopoulos 1970*a*, 1972). The reaction of the gas can be calculated from the magnetohydrodynamic equations, in which the peculiar motions of the H I clouds are usually simulated by a pressure term (see e.g., Roberts and Yuan 1970). Analytically, the response problem can be solved for tightly wound spirals in an asymptotic approximation (e.g., Lin et al. 1969). This procedure corresponds to the WKB method of quantum mechanics and is based primarily on the assumption that $\Phi(R)$ varies so rapidly with R that the wavelength λ is small compared with R. Numerically, the response problem can also be handled for open spirals (Kalnajs 1970). Lin and his co-workers come to the following results for the density wave induced by V_1. For a tightly wound spiral, the density wave is essentially in phase with the potential wave, i.e., the minima of V_1 correspond to maxima of the density perturbation σ_1:

$$\sigma_1 = -\hat{\sigma}_1(R)\, e^{i(\omega t - m\theta + \Phi(R))} \quad . \tag{14}$$

The relative amplitude of the density wave, $\hat{\sigma}_1/\sigma_0$, is greater, the smaller the velocity dispersion of the objects is, because peculiar velocities tend to smear out local inhomogeneities. Therefore, the amplitude of the density wave induced by V_1 is relatively larger in the gas than for the bulk of the stars, because of the different velocity dispersions of the gas and the stars. Quantitatively, the leveling effect of the velocity dispersion is derived from so-called 'reduction factors' F, which describe how much the response of an actual population of stars or gas is smaller than that of a hypothetical population with zero-velocity dispersion. Typical local values are $F = 0.45$ for the gas and $F = 0.066$ for the stars (Lin 1970*b*).

The perturbations in the mean velocities of the stars and the gas are also derived: the velocity perturbation $v_{R,1}$ in the radial direction is in antiphase with the density response, i.e., the largest inward streaming occurs at the highest density (center of an arm); the velocity pertur-

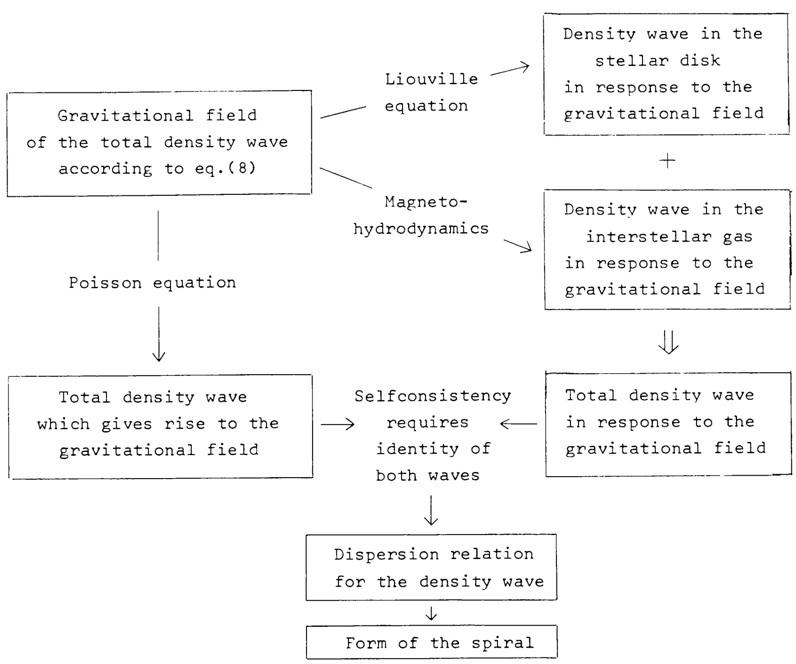

FIG. 5 — Lin's solution scheme.

bation $v_{\theta,1}$ in the tangential motion is shifted by 90° in phase with respect to the density variation, so that the most rapid rotation occurs at the outside of a spiral arm. This is just the behavior which is expected from the kinematical density waves discussed in section VI. The ratio of the amplitudes in σ_1, $v_{R,1}$, $v_{\theta,1}$ is approximately given by

$$(\hat{\sigma}_1/\sigma_0):(\hat{v}_{R,1}/v_{\rm rot}):(\hat{v}_{\theta,1}/v_{\rm rot}) = 1: \left|\frac{\lambda}{\pi R}\left(1-\frac{\Omega_p}{\Omega}\right)\right| : \left|\frac{\lambda\kappa^2}{4\pi R\Omega^2}\right| \quad .(15)$$

Using for $R = 10$ kpc the values $\lambda = 4$ kpc ($\psi \sim 7°$), $\Omega_p/\Omega \sim 1/2$ and $(\kappa/\Omega)^2 = 1.6$, we find the ratio 1 : 0.064 : 0.051. Assuming for the stars an amplitude $\hat{\sigma}_1{}^*$ of 5% of the unperturbed density $\sigma_0{}^*$, we predict velocity amplitudes $\hat{v}^*_{R,1}$ and $\hat{v}^*_{\theta,1}$ of 0.8 and 0.6 km sec^{-1}. For the gas, where the relative density contrast is much larger, $\hat{\sigma}_1{}^g \sim 50\%\ \sigma_0{}^g$, the corresponding amplitudes in the systematic deviation from circular rotation should be of the order of 8 and 6 km sec^{-1}. The nonlinear response of the interstellar gas and the occurrence of shocks will be discussed in section XIII.

Having discussed the response problem, we shall now calculate, according to Figure 5, the density wave that is causing the assumed perturbation V_1 in the potential. This is done by solving Poisson's equation. For Lin's mode of a tightly wound spiral the asymptotic approximation ($\lambda \ll R$) leads to a simple local relation

$$V_1 = -\lambda G\sigma_1 \quad , \qquad (16)$$

between the perturbation V_1 in the potential and the density wave σ_1 causing it. Hence we find again that V_1 and σ_1 are in (anti) phase for Lin's mode. Since galaxies are almost isolated systems, we must require self-consistency: the density wave causing V_1 according to equation (16) must be identical with the sum of the density waves that were created in the stellar population and in the gas as a response to V_1. In the general case of density waves, e.g., for open spirals, this identity has the form of an integral equation (Kalnajs 1971; Contopoulos 1973*a*), because the potential wave V_1 depends on the global structure of the density wave σ_1 and vice versa. Such an integral equation can be conveniently solved only by numerical techniques (Kalnajs 1970).

The equation for the identity of cause and effect found locally in the case of Lin's mode for tightly wound spirals, is called the 'dispersion relation'. It gives a connection between the local wavelength $\lambda(R)$ and the local relative frequency $\nu(R)$ of the wave. The frequency ν,

$$\nu = m(\Omega_p - \Omega)/\kappa \quad , \qquad (17)$$

describes the frequency of passage of spiral arms past an observer who corotates with the material, measured in units of the epicyclic frequency. The form of the dispersion relation $\lambda(\nu)$ is shown in Figure 6. The wavelength λ is measured in a local unit λ_*

$$\lambda_* = 4\pi^2 G\sigma/\kappa^2 \quad . \qquad (18)$$

Because the density response depends on the velocity dispersion in the galaxy, the dispersion relation contains the relative velocity dispersion

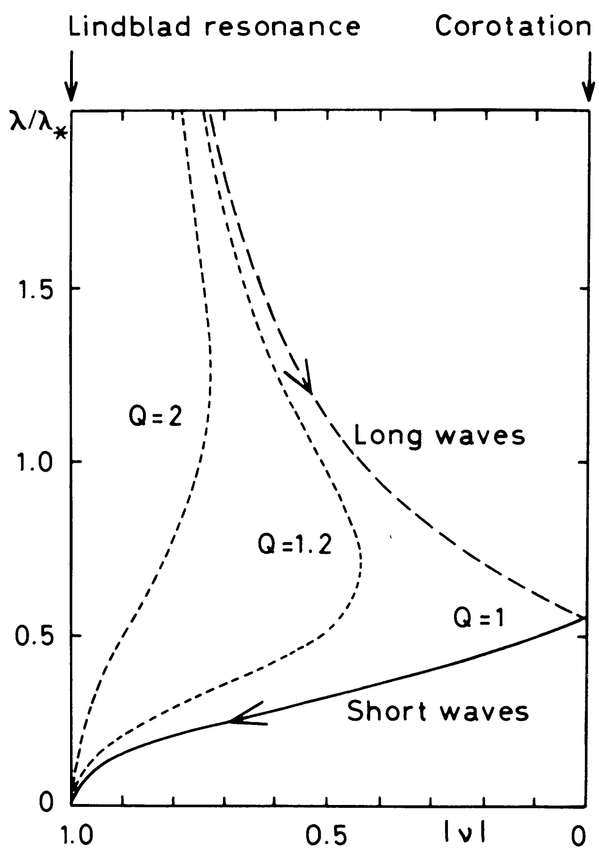

FIG. 6 — Dispersion relation $\lambda(\nu)$ for density waves (Lin and Shu). The arrows show the direction of propagation of the waves. Q measures the velocity dispersion.

Q, i.e., the ratio between the actual and the critical dispersion (section VII), as a parameter. Lin et al. usually assume marginal stability for galaxies, i.e., $Q = 1$. In this case, solutions for density waves of Lin's type exist for $\nu^2 < 1$, i.e., $-1 < \nu < +1$. According to the definition of ν (eq. 17), this interval corresponds to

$$\Omega - (\kappa/m) < \Omega_p < \Omega + (\kappa/m) \quad . \qquad (19)$$

The density waves must therefore be contained within the two circles for which R fulfills the condition $\Omega_p = \Omega \pm (\kappa/m)$. For a two-armed spiral, $m = 2$, the density wave can extend over the region between the inner and outer Lindblad resonance. For any given value of Ω_p, these limiting radii can be read off from Figure 2 where $\Omega - (\kappa/2)$ and $\Omega + (\kappa/2)$ are given as a function of R for our Galaxy. For values of m higher than two, the permitted range in R shrinks very much. Thus Lin's theory can explain the preference of two-armed spirals for the grand design, because only such waves can cover simultaneously a large region of a galaxy. If Q is significantly larger than one, then the region around the corotation radius ($\nu = 0$, $\Omega = \Omega_p$) is forbidden for density waves. This would lead to two separated and more narrow regions in which density waves are allowed to exist. According to Contopoulos (1973*a*), this uncomfortable result for $Q > 1$ may disappear in a more elaborate treatment of the corotation resonance region.

Lin's dispersion relation (Fig. 6) gives for each permitted value of ν two solutions for λ: short and long waves. Lin et al. consider mainly the short waves, i.e., tight spirals. The long waves are not really consistent with the asymptotic approximation used, since the condition $\lambda \ll R$ is not fulfilled for these long waves. Hence the problem of the coexistence of short and long density waves with the same value of Ω_p is not completely settled.

The linear treatment of density waves breaks down at the Linblad resonances ($\nu = \pm 1$) and partly also at the corotation resonance ($\nu = 0$). Formally, the linear theory leads for $|\nu| \to 1$ to $\lambda \to 0$ and $\hat{\sigma}_1 \to \infty$, which is unrealistic. In the neighborhood of the resonances, special methods must be applied (see e.g., Contopoulos 1973*a*; Mark 1971). At the resonances, the density response is not any more in phase with the wave in the potential. A nonlinear treatment of the orbital behavior of stars near the resonances (Contopoulos 1970*b*, 1973*a,b*) has shown that, near the inner Lindblad resonance, the stars are concentrated in two oval rings around two perpendicular, elongated, periodic tube orbits. At the corotation resonance, stars get trapped in two banana-like regions around two Lagrangian points (similar to Trojan asteroids). Under these conditions, however, a self-consistent solution is difficult to obtain, and the problem of saturation effects become severe. This is rather unfortunate, because the resonances seem to be very important for the exitation and maintenance of density waves (section XII).

The amplitude of the density wave remains undetermined in the linear theory. In a higher approximation, Shu (1970*b*) derived the relative dependence of the amplitude $\hat{\sigma}_1$ as a function of R, but $\hat{\sigma}_1(R)$ still contains an arbitrary factor. Even this improved treatment leads to singularities in the amplitudes at the Lindblad and corotation resonances. By considering the absorption of a density wave by resonant stars, Mark (1971) found that the amplitude of the wave attains a finite maximum slightly outside the inner Lindblad resonance, but is evanescent at and inside the resonance. A complete solution of the amplitude problem can only be provided by nonlinear investigations of density waves. Some progress on the nonlinear treatment has been achieved by Vandervoort (1971) and Aoki (1974).

The linear theory of density waves can explain trailing and leading spirals equally well. Away from resonances, even nonlinear calculations have so far been unable to demonstrate a preference for trailing spirals, contrary to initial expectation. At present, mainly the behavior of the wave at the inner Lindblad resonance (Contopoulos 1971; Mark 1971) favors trailing spiral arms. Of course, the generation mechanisms for density waves (section XII) may be mainly responsible for the trailing of spiral arms, observed in real galaxies.

X. Resulting Spiral Structure

Using the dispersion relation $\lambda(\nu)$ for density waves, the form of the spiral structure can be derived. There is one free parameter in Lin's theory, namely the rotation frequency Ω_p of the wave. For a given value of Ω_p and a given mass model of a galaxy with $Q = 1$, the form of the spiral is fixed uniquely in Lin's theory. From the quantities $\Omega(R)$, $\kappa(R)$, $m = 2$, and Ω_p, the frequency $\nu(R)$ of the wave can be calculated from equation (17). The dispersion relation $\lambda(\nu)$, shown in Figure 6, then gives $\lambda(R)$. By numerical integration, the radial phase $\Phi(R)$ of the wave follows from $\lambda(R)$ according to equation (13). Knowing $\Phi(R)$, the geometric form of the spiral is obtained from equation (11). The resulting spiral structure of our Galaxy is shown in Figure 7 on the assumption that $\Omega_p = 11$ km sec^{-1} kpc^{-1} (Lin et al. 1969). Here Ω_p and the zero point of the phase Φ have been chosen in such a way as to fit as well as possible the spiral structure in the H I gas, determined from 21-cm observations. This fitting is rather difficult, since different groups of radio astronomers have derived conflicting results on the spiral structure of our Galaxy (Kerr 1969, 1970; Weaver 1970; Simonson 1970; Bok 1971, 1972). The Perseus and Sagittarius spiral arms fit into Lin's picture, but not the local Orion arm. According to Lin et al., the Orion arm should be a local phenomenon, like a bridge between the main arms. It must be emphasized here that the Lin theory sets out primarily to explain the grand design of spiral structure and is quite willing to allow the coexistence of secondary density waves and local

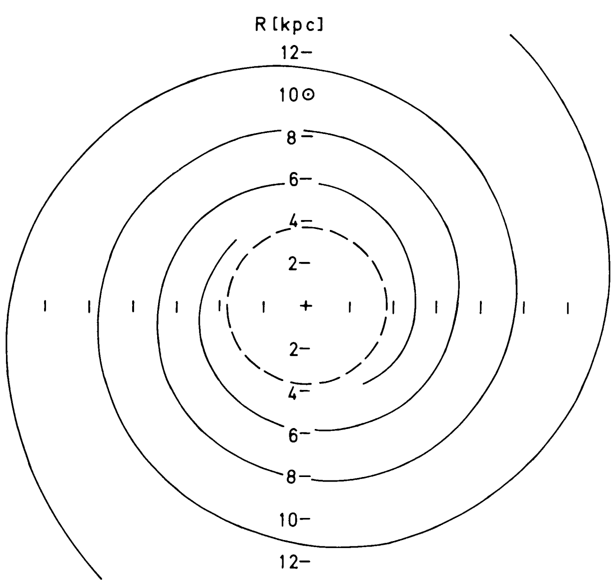

FIG. 7 — Spiral structure of our Galaxy according to Lin et al. for $\Omega_p = 11$ km sec^{-1} kpc^{-1}.

material arms along with the primary structure. The theory agrees with the observations in that inside the inner Lindblad resonance there are no spiral arms. Lin associates the observed '3-kpc arm' with the inner Lindblad resonance. Further properties of the density wave in our Galaxy are given in Table I. According to Lin et al., the stars and the gas contribute approximately equal amounts to the wave in the surface density. Since the gas makes up only a small fraction of the total surface density σ, the density wave in the gas is very pronounced while the density wave in the stellar population remains relatively small. The absolute value of the amplitude of the density wave cannot be derived from the present theory, but must be found by fitting observational features, such as the density contrast in the H I gas, the amplitude of the 'waves' in the apparent rotation curve, and the places of formation of stars (Barbanis and Woltjer 1967; Yuan 1969*a*,*b*).

TABLE I

BASIC PROPERTIES OF THE DENSITY WAVE IN THE MILKY WAY ACCORDING TO LIN

Angular speed Ω_p of the spiral pattern	13.5 km sec^{-1} kpc^{-1}	
Inner Lindblad resonance	3.2 kpc	
Corotation	15.8 kpc	
Outer Lindblad resonance	21 kpc	
At $R \sim 10$ kpc:		
Local frequency ν	-0.73	
Local period $\pi/(\Omega - \Omega_p)$	2.7×10^8 years	
Local wavelength λ	~ 4 kpc	
Amplitude of the gravitational field of the wave	5% of the average field ($\Omega^2 R$)	
Amplitude $\hat{\sigma}_1$ of the wave in the surface density		
total	10%	of the average total surface density σ_0
stars and gas each	5%	of the average total surface density σ_0
Amplitude $\hat{v}_{R,1}$ of the deviation of the mean motion from circular motion		
stars	$\lesssim 1$ km sec^{-1}	
gas	$\lesssim 10$ km sec^{-1}	

The value for Ω_p derived by Lin and his co-workers, namely, $\Omega_p \sim 13.5$ km sec^{-1} kpc^{-1}, is only slightly higher than the value of 8 km sec^{-1} kpc^{-1} for which an almost global resonance (section VI) would occur. Lin's value of Ω_p also means that the density wave rotates only about half as fast as the material at the sun's distance ($R = 10$ kpc) from the galactic center ($\Omega = 25$ km sec^{-1} kpc^{-1}). Hence the relative velocity between the mean motion of the stars and gas and the density wave is about 115 km sec^{-1} in our neighborhood. Other authors (Kalnajs 1970; Marochnik and Suchkov 1969*a,b,c*; Marochnik, Mishurov, and Suchkov 1972) have advocated higher values of Ω_p for our Galaxy, up to about 30 km sec^{-1} kpc^{-1}. Such a high value of Ω_p leads to a very open spiral. However, Kalnajs (1970) has shown that the response of the gas to such an open spiral potential, provided by the stars, is actually a tightly wound pattern. This shows that the spiral structure observed in the gas and young stars is not necessarily representative for the form of the overall density wave. It is only true for tightly wound spirals, treated by Lin in the asymptotic approximation, that the response of both the gas and the stars are essentially in phase with each other and with the wave in the potential.

Shu et al. (1971) have applied the Lin theory to the external galaxies M33, M51, and M81. They first construct mass models for each of these galaxies from the observed rotation curves, in order to have available all relevant quantities such as $\Omega(R)$, $\kappa(R)$, $\sigma(R)$. They derive the velocity dispersion $v_{\rm rad}(R)$ by assuming marginal stability ($Q = 1$). The pattern speed Ω_p is chosen for each galaxy in such a way that the density wave corotates with the outermost H II regions. The motive for this choice of Ω_p will be discussed in section XII C. They obtain $\Omega_p = 16$ km sec^{-1} kpc^{-1} for M33 (corotation at $R_c = 6.8$ kpc), 33 km sec^{-1} kpc^{-1} for M51 ($R_c = 4.5$ kpc), and 21.5 km sec^{-1} kpc^{-1} for M81 ($R_c = 11.2$ kpc). By using these data and the dispersion relation according to Lin's theory, Shu et al. succeed in reproducing the observed form of spiral structure in the three galaxies. M33 rotates almost rigidly and has no inner Lindblad resonance. Therefore the spiral structure reaches to the center of M33. It is remarkable that Lin's theory can reproduce even such an open spiral like M33, in spite of the asymptotic approximation ($\lambda << R$) used in the theory. M81 is in strong differential rotation. The inner Lindblad resonance is predicted at $R = 1.7$ kpc. In agreement with the theoretical predictions, no spiral structure is observed inside of that circle and tightly wound arms begin there. The galaxy M51 also rotates strongly differentially and its inner Lindblad resonance occurs at $R \sim 1$ kpc. The spiral is moderately tight. Lin's theory can reproduce the arms except in the outermost region, which is strongly disturbed by the companion NGC 5195 (Toomre and Toomre 1972). In M51, Shu et al. even find indications of the existence of the long waves of Lin's mode in the form of a weak, wide-open spiral being superimposed on the main arms which are made by the short waves (schematically indicated in Fig. 8).

XI. Objections Against Lin's Mode

No observations have so far contradicted Lin's interpretation of spiral structure as a special form of density waves (Lin's mode), though Piddington (1973*a,b*) claims to have evidence against Lin's density-wave theory. Lin's density-wave theory is, however, confronted with some theoretical difficulties:

A. *Antispiral Theorem.* This theorem states that in the linear approximation, neutral spiral density waves are possible neither in the gas (Lynden-Bell and Ostriker 1967) nor among the stars (Shu 1970*a*), if no resonances and no dissipation are present. The antispiral theorem would allow neutral waves of the wagon-spoke type with constant radial phase Φ only. There are various reasons why the antispiral theorem may not be applicable to real galaxies. First, in galaxies resonances and dissipation certainly do exist. Although these resonances and dissipative effects are indeed not explicitly considered in the treatment of Lin's mode, it is conceivable that the actual resonances and dissipation might allow the existence of neutral spiral waves in some indirect manner. In fact, the resonant stars seem to be very important for the excitation and maintenance of density waves. Second, the antispiral objection disappears if the density wave is not strictly neutral but is weakly damped or growing (ω complex). Such a situation ($\omega_I \neq 0$) is highly probable for other reasons, discussed under point C.

B. *Why Only One Frequency* Ω_p? Lin considers, out of the continuous Ω_p spectrum allowed within the frame of his approximation, just one frequency Ω_p. The most general solution should be a superposition of all permitted frequencies. The resultant density wave would then not rotate rigidly but would disperse instead of providing a permanent spiral structure. However, a more elaborate treatment of density waves taking into account proper boundary conditions, is expected to reveal some discrete, isolated eigenvalues for ω and hence for Ω_p. The calculation of such eigenvalues is difficult and uncertain, because the resonances occur just at the boundaries. Kalnajs (1970) has searched for the fastest growing mode of two-armed density waves in a galaxy similar to our Milky Way, and finds an eigenvalue for this mode corresponding to $\Omega_p \sim 30$ km sec^{-1} kpc^{-1} with an exponential growth time of about 3×10^8 years. Of course, such a result does not necessarily contradict Lin's value of $\Omega_p = 13.5$ km sec^{-1} kpc^{-1}. First, actual density waves with finite amplitudes may favor other eigenvalues than those predicted from infinitesimal linear waves, because of various nonlinear effects. Second, the density waves may be excited by an imposed field, such as that of a rotating bar at the center of a galaxy. Then, Ω_p is essentially determined by this excitation mechanism.

C. *Group Velocity of Density Waves.* Toomre (1969) has shown that density waves of Lin's type necessarily have a group velocity v_g different from zero. Any packet of density waves propagates radially and may finally disappear. The energy E of the wave or, more strictly, its action density $E/(\Omega - \Omega_p)$ is transported radially away from the corotation resonance toward the Lindblad resonances for trailing, short waves of Lin's type (see also Dewar 1972; Hunter 1973). For the long waves of Lin's mode, the direction of propagation is in the opposite sense, i.e., toward the corotation resonance (Shu 1970*b*). Furthermore, Toomre finds that a packet of spiral density waves may be wound up in the course of time. These results indicate that density waves cannot be permanently neutral waves without some energy replenishment. For the radial group velocity of Lin's density wave at $R = 10$ kpc, Toomre finds $v_g \sim -10$ km sec^{-1}. Hence the density wave would disappear after a time of about 10 kpc/10 km sec$^{-1} \sim 10^9$ years, if no regeneration of the wave takes place.

XII. Origin of Density Waves

The rather short lifetime of a (free) density wave, found by Toomre (1969), and the dissipation time of the density wave due to the shock in the gas, discussed later in section XIII, forces the theory to assume either a continuous energetic regeneration of a once-existing density wave or else a frequent generation of new density waves, at intervals of at most 10^9 years. There have been essentially six generation mechanisms for density waves proposed so far. Of course, different mechanisms may be responsible first for the creation and later for the maintenance of spiral structure.

A. *Influence of Neighboring Galaxies.* Permanent companions, or accidentally passing galaxies, are able not only to bend the plane of disk-shaped galaxies, but they can also create density waves within the plane (Toomre 1969, 1970). At first, the tidal interaction between two galaxies will produce trailing spiral arms in the outer regions of a galaxy (see e.g., Pfleiderer 1963; Toomre and Toomre 1972). For our Galaxy, the Large Magellanic Cloud could be the source of such tidal arms. These outer material arms will then excite a density wave which propagates inward with the group velocity v_g, thus producing spiral structure also in the inner regions of the galaxy. These density waves would be a transient phenomenon, lasting about 10^9 years. Although tidal interaction is a promising mechanism for generating spiral density waves in some galaxies with suitable companions, it is hardly conceivable that *all* spirals are produced in such a manner, because of the many rather isolated spiral galaxies.

B. *Central Asymmetry.* Many S-type galaxies show an elongated structure in their central regions, and a continuous transition from S to SB galaxies seems probable (e.g., de Vaucouleurs 1959; Freeman 1970). A small central bar or an oval distortion of the inner region is capable of exciting and maintaining density waves in the outer regions of a galaxy. Feldman and Lin (1973) have found that an oval distortion of the central regions forces a trailing spiral wave near the corotation circle. Such a driven wave could then propagate inwards, causing a permanent spiral structure over a large portion of a galaxy. An oval distortion with an axial ratio of $b/a = 0.9$ produces a spiral field of a few percent of the axisymmetric field. This explanation for the origin and maintenance of density waves is very attractive, but merely shifts the problem into the question of how the inner asymmetry originates. Of course, it may be just the density wave itself which creates some oval distortion of a galaxy near the inner Lindblad resonance (Contopoulos 1970*b*, 1973*a*). But probably it is a more fundamental, global instability of galactic disks which produce such asymmetric distortions of flat galaxies (Hohl 1971*a*; Kalnajs 1972*a*).

C. *Local Gravitational Instabilities.* Lin (1970*a*, 1971) has proposed that density waves are caused by Jeans instabilities in the H I gas in the outer regions of galaxies. As the observations show, the gas layer of a galaxy usually reaches considerably farther out than the stellar disk. Because of the small velocity dispersion of the gas and the lack of stabilization by stars, the gas becomes gravitationally unstable in those outer regions. These gaseous condensations are stretched out into trailing material arms by differential rotation and can produce growing 'wavelets' in the gas (Goldreich and Lynden-Bell 1965) and also in the stellar disk (Julian and Toomre 1966). Thereby, a tightly wound, trailing density wave may be excited at the corotation region, and will propagate then mainly toward the inner Lindblad resonance. The frequency Ω_p of this density wave should agree approximately with the rotational frequency in the outer region of a galaxy. The values of Ω_p which allow explanation of the observed spiral structure (section X), support this prediction. At the inner Lindblad resonance, the inward-moving short waves (tight spiral) should be transformed, by some yet unspecified nonlinear effects into long waves (open spiral), which then run outwards. Figure 8 shows this schematically. The outward-moving long waves run back into the original arms at the corotation circle and produce an energetic feedback, thus maintaining the whole density wave (Lin 1970*a*). Alternatively, the absorption of the short waves produce the oval distortion of the galaxy near the inner Lindblad resonance, and this asymmetry directly regenerates the wave at corotation (Feldman and Lin 1973), thus also providing a feedback.

D. *Angular Momentum Transfer.* Lynden-Bell (1970), Kato (1971), Lynden-Bell and Kalnajs (1972), and Kalnajs (1973*b*) have proposed that the density waves in galaxies are excited and maintained because the waves transfer angular momentum and energy from the inner part of a galaxy to the outer. This outward transfer would lower the overall rotational energy of a galaxy, if the spiral arms are trailing. The energy for maintaining density waves would be taken from the basic rotation of a galaxy. If we postulate a tendency for galaxies to evolve in the direction of increasing entropy and hence toward lower rotational energy, the density waves may be just the instrument by which galaxies try to follow such a 'thermodynamical trend'. The details of this generating mechanism for density waves are still somewhat uncertain (Kalnajs 1973*b*). In any case, we have to realize that the density waves will lead to a slow dynamical evolution of spiral galaxies, since the radial distribution of angular momentum in the galaxy can be changed because of the gravitational torques produced by the spiral wave.

E. *Two-Stream Instability.* The two-stream instability is well known in plasma physics. A similar instability may occur in stellar systems and could excite density waves. Marochnik and

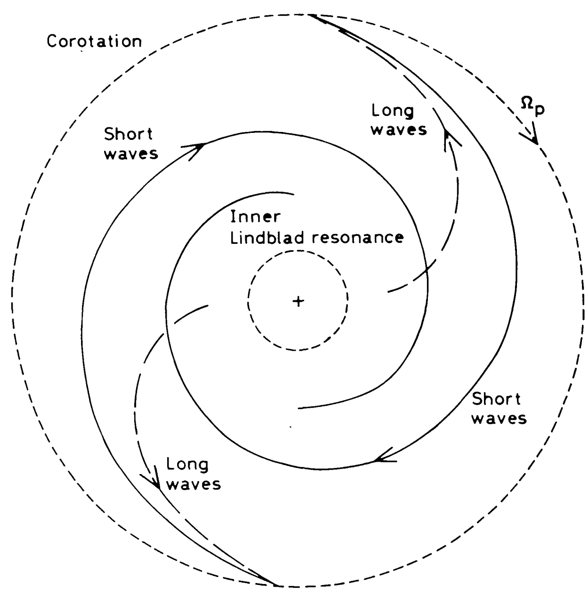

FIG. 8 — Schematic spiral structure of a galaxy with superposition of short and long waves (Shu et al.).

Suchkov (1969*a,b,c*), Marochnik et al. (1972), Harrison (1970), Hohl (1971*c*) and Kato (1973) have studied such possibilities for relative motions either between two stellar populations or between the stars and the gas. The results are somewhat inconclusive because of various simplifications and assumptions involved.

F. *Eruptive Activity of the Nucleus.* The production of spiral arms by large-scale expulsions of gas from the galactic nucleus was first proposed by Ambartsumian (e.g., 1958), later emphasized by Arp (1969*a,b*) and recently discussed by Oort (1973*a,b*) and his coworkers (van der Kruit, Oort, and Mathewson 1972; van der Kruit 1973*a,b*). It is quite uncertain whether such a process can excite the density waves required for a grand design.

It is certainly the largest problem for the density-wave theory at present that we do not have a convincing answer to the question how the density waves are actually excited and maintained in most galaxies.

XIII. Shock Waves in the Gas

In the linear approximation, the density waves in the gas and in the stellar disk mainly differ through their relative amplitudes. Nonlinear calculations, however, show a qualitatively new effect, namely the appearance of a shock front in the gas. Such galactic shock waves were first proposed by Fujimoto (1968*a,b*). Roberts (1969) developed the theory of shock waves in the presence of a spiral potential of Lin's type. Reviews on the density-wave theory with special emphasis on galactic shocks have been published by Roberts (1974) and Shu (1973*a*).

It is already indicated in the linear theory that the response of the interstellar gas even to a small background spiral gravitational field should be rather large, because of the small velocity dispersion in the gas. If the amplitude of the streaming velocity of the gas induced by the spiral field, becomes significantly larger than the turbulent velocities in the gas ($\sim$8 km sec^{-1}), then the gas flow must be partly 'supersonic'. At the transition region from supersonic to subsonic velocities, a shock front occurs. Roberts (1969) has calculated the properties of a two-armed spiral shock pattern for Lin's mode of density waves. The main results are shown in Figures 9, 10, and 11. Instead of sinusoidal waves in the density and mean velocity, now discontinuities appear at the shock front (Fig. 9). At $R \sim 10$ kpc, the maximum density in the shock front is about 5 times the mean density, and the ratio between the densities just before and after the shock front is about 1 : 8. The streamlines of the gas are stationary, closed curves in the rotating Ω_p system, but they show a bend at the shock front (Fig. 10). The gas flows into the spiral arms supersonically from behind, with an appreciable angle to the arm, is slowed down in the shock front to a subsonic velocity, and leaves the spiral arm slowly and almost tangentially. After an acceleration phase, the gas reaches the next arm, and the whole process is repeated cyclically.

The sudden increase in the density and pressure of the interstellar gas on arrival at the shock front will create an external compression of the H I clouds. In this way, the gravitational collapse of interstellar clouds may be started, because some clouds were probably not very far from instability before. A cloud stays for about 3×10^7 years in the domain of high compression. In this time, a considerable number of clouds can collapse, and through consequent fragmentation star formation will occur. Since the shock front is located on the inner edge of the H I arm, the youngest O stars and the associated H II regions will occur there (Fig. 11). Hence galactic shock waves, viewed as a triggering mechanism for the formation of bright, young stars and H II regions, provide an ample explanation for the narrow spiral arms which are optically most prominent

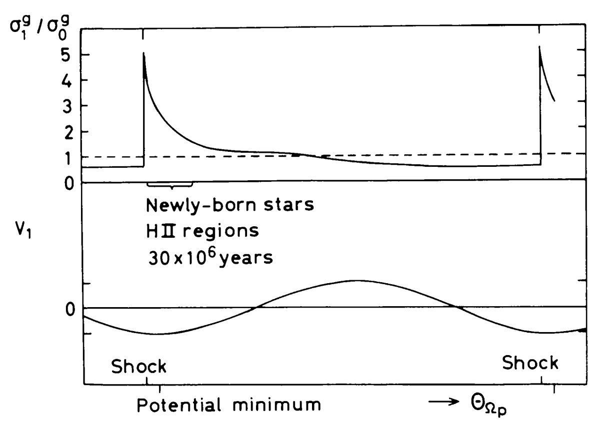

FIG. 9—Variation of the relative gas density $\sigma_1{}^g/\sigma_0{}^g$ along (half of) a streamline for $R \sim 10$ kpc (W. W. Roberts). V_1 is the adopted wave in the potential.

in many spiral galaxies. The shock front does not only collect already existing dust particles but may also favor the formation of new dust particles because of the increased gas density (see also Shu 1973*b*). Thus the shock wave may also explain the striking absorption lanes on the inner edges of spiral arms (Lynds 1970, 1972). A more realistic treatment of the shock waves in the interstellar medium must take into account the existence of two phases of the interstellar medium, namely cool, dense clouds, embedded into a hot, rarefied intercloud gas. Such detailed investigations, including phase transitions triggered by the spiral density wave, have been carried out by Pikelner (1970*a,b*) Biermann et al. (1972), and Shu et al. (1972).

For the magnetic field frozen into the gas there will be no winding-up problem if the magnetic-field lines agree with the closed streamlines of the gas. This assumption allows a magnetic field which is stationary in the Ω_p system. The local strength seen by an observer comoving with the gas would vary periodically according to the density variation along a streamline. Therefore, the field strength in the spiral arms should be significantly higher than outside of an arm. Furthermore, the magnetic field in the spiral arm should be in general parallel to the arm (see Fig. 10), in agreement with the observations. According to Roberts and Yuan (1970) a magnetic field of $5 \mu G$ in the arm would reduce the maximum compression factor at $R \sim 10$ kpc from 5 to 3, and would therefore not affect the triggering mechanism for star formation appreciably. A magnetic field of 20 μG or more, however, would not allow a sufficient compression of clouds. In all cases, the magnetic field decreases the amplitude of the density wave in the interstellar gas, especially the systematic deviations of the gas velocity from pure circular motion.

Tosa (1973) has studied three-dimensional galactic shock waves, taking into account the finite thickness of the gas layer in the z-direction. He finds that the effective thickness varies along a streamline. At the shock front, the thick-

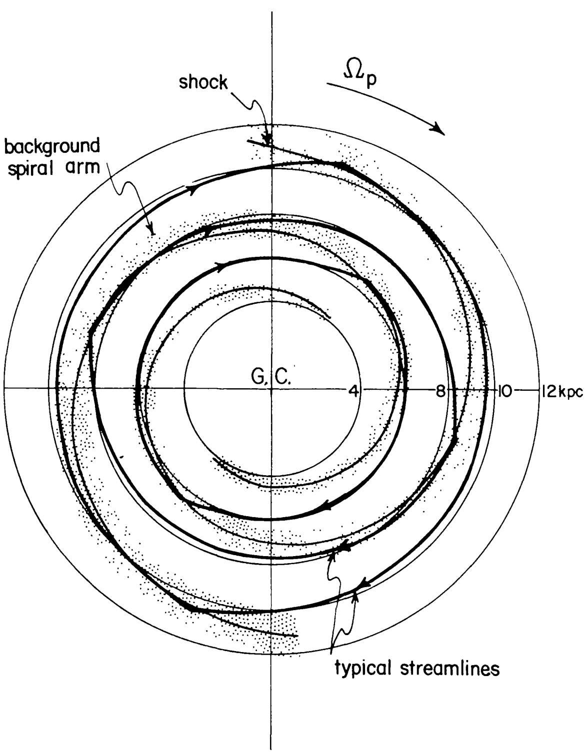

Fig. 10 — Typical streamlines of the gas in the Ω_p system (W. W. Roberts). The wave in the potential is shown by shading. The location of the shock front is also sketched in.

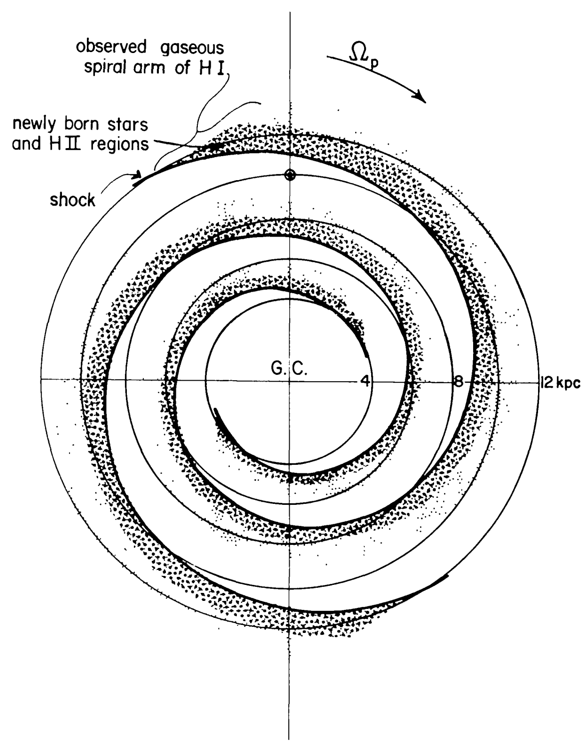

Fig. 11 — The appearance of a spiral galaxy according to W. W. Roberts, with the spatial sequence of shock front, youngest stars, and gaseous arm.

ness increases by 30%–40%. This decreases the gas compression slightly. Furthermore, the sudden expansion in the z-direction at the shock front may lead to an expulsion of gas. This could explain high velocity H I gas, observed far above the spiral arms. Schmidt-Kaler and House (1974) have investigated density waves in the z-direction in order to explain a filamentary structure ('shingles') of spiral arms.

Is there any direct observational evidence for galactic shock waves in the gas? In our Galaxy, it should be very difficult to detect the shock front in the 21-cm profiles for various reasons. Somewhat indirect evidence for a shock wave in the Perseus arm has been found by Roberts (1972). Probably the most suggestive evidence for shock waves comes from the synchrotron radiation of spiral arms, observed at 1415 kHz in external galaxies by using large synthesis radiotelescopes (Mathewson et al. 1972; van der Kruit 1973*b*). The synchrotron radiation is strongly concentrated in a narrow lane at the inner side of the optical spiral arm (good example: M51). This can be explained by the shock front which should have such a position relative to the optical and H I arms: The shock wave compresses the interstellar gas, therefore the frozen-in magnetic field increases there, and because the synchrotron emissivity goes up with nearly the square of the magnetic field strength, the strongest synchrotron radiation originates in a narrow zone behind the shock front.

How large must the amplitude of the spiral gravitational field be in order to produce a shock wave in the interstellar gas? Shu, Milione, and Roberts (1973) have concluded that galactic shocks must arise necessarily, if the strength of the spiral gravitational field exceeds a certain critical value, which corresponds to a relative amplitude in the radial force of slightly more than 1%. For a moderate field strength ($\sim$2%), there occur additional shock waves between the main arms, because of resonance effects. This would lead to a four-armed spiral shock pattern, and may explain some of the spiral arms observed in our Galaxy, e.g. the Carina arm or even the local Orion arm, which otherwise do not fit into the grand design of Lin's theory. A larger field strength ($\sim$5% and more) produces two-armed galactic shocks, while spiral gravitational fields below the critical value cause merely smooth, slightly nonlinear waves without a shock front, similar to the waves studied by Vandervoort (1971). The actual relative strength of the spiral gravitational field can vary from galaxy to galaxy: Galaxies with a shock wave would show narrow spiral arms (e.g., M81), whereas those without a shock wave would have quite broad spiral arms (e.g., M33). The strength of the shock in a galaxy seems also to provide an explanation for van den Bergh's (1960*a,b*) luminosity classification of galaxies, which is based on the regularity and brightness of the optical spiral arms: Roberts, Roberts, and Shu (1974) investigated 25 galaxies and find a strong correlation between the predicted strength of the shock and the observed luminosity class of a galaxy (see also van der Kruit 1973*b,c*).

The shock wave in the interstellar gas causes some difficulties for the density-wave theory. First, it is technically difficult to obtain a completely self-consistent solution for the overall density wave in the presence of a shock wave in the gas. In fact, no really self-consistent solution has been calculated up to now, only the response problem has been solved. It is supposed (e.g., Roberts and Yuan 1970) that no severe modifications of the present picture would emerge from a self-consistent treatment. Second, the shock involves dissipative effects and will therefore damp the overall density wave. For the same reason, the streamlines of the gas cannot exactly close upon themselves. Kalnajs (1972*b*) has derived a very short damping time for the density wave of Lin's type because of the shock, namely about 10^8 years. Roberts and Shu (1972) have modified Kalnajs' calculations and arrive at $6–9 \times 10^8$ years for the damping time. This is comparable to the wave-propagation time of 10^9 years, based on the group velocity of the wave (section XI C). The rapid damping of a density wave by the induced shocks in the gas accentuate the necessity of a permanent regeneration of density waves (section XII).

XIV. Some Applications

The quantitative results developed by Lin and his coworkers for the structure of spiral density waves in galaxies, especially in our Milky Way, allow many applications to other problems of galactic research. We will list here only a few examples.

A. *'Waves' in the Rotation Curve.* The rotation curve of our Galaxy shows irregularities of wavy structure, if one determines the rotational velocity $v_{\mathrm{rot}}(R)$ from observations of 21-cm profiles under the assumption of pure circular motion of H I by applying the tangential-point method (Shane and Bieger-Smith 1966). The density-wave theory can satisfactorily explain the amplitude and position of these apparent bumps in the rotation curve through the systematic motions in the gas (Yuan 1969*a*). Since the gas does not move on circular orbits, the maximum radial velocity along a line of sight does not occur at the tangential point nor is the gas velocity there identical with the circular velocity. However, the well-known north-south asymmetry of the rotation curve cannot be easily explained by density waves and suggests rather an oval vibration mode for galaxies (Hunter 1963, 1965, 1970).

B. *Evaluation of* H I *Densities.* The interpretation of the observed 21-cm profiles of H I must take into account the probable existence of density waves. If the structure of the density wave is given, then one can calculate for each line of sight the relation between radial velocity and distance of the gas. Unfortunately, this relation is much more complicated than for circular orbits. Therefore, a simple assignment of the maxima in the 21-cm profiles to density maxima (spiral arms) is not possible any longer. It is more convenient to start from a model of the distribution and kinematics of the H I gas, to construct the profiles that are to be expected from the model, to compare these with the observed profiles (Yuan 1969*a*; Yuan and Liebovitch 1972; Burton and Shane 1970; Burton 1971, 1972; Shane 1972), and then to improve the model iteratively. The major problem is to reach an at least kinematically self-consistent model for the density and velocity distribution in the presence of a density wave.

C. *Distribution of* H II *Regions.* Radio observations of giant H II regions show that most of these H II regions occur in a ring between 4 kpc and 8 kpc from the galactic center (Mezger 1970). On the other hand, the surface density of neutral hydrogen (H I) attains its maximum between 8 kpc and 14 kpc. The abundance distribution of H II relative to that of H I as a function of R is in rather good agreement with the predictions of the density-wave theory (Lin 1971; Mark 1971; Shu 1973*a*). The scarcity of giant H II regions for $R < 4$ kpc is due to the absence of a density wave, and hence of a shock wave and of star formation, inside the inner Lindblad resonance. For $R > 4$ kpc, the rapid decrease outward of the H II/H I ratio can be attributed to a decreasing rate of star formation with increasing R, because (a) the shock strength (peak gas compression) decreases outward and (b) the frequency with which the streaming gas encounters shock fronts, $2(\Omega - \Omega_p)$, decreases outwards too. A similar explanation for the radial distribution of neutral hydrogen in galaxies has been given by Oort (1974).

D. *Birthplaces of Stars.* The places of formation of stars can be determined by tracing backwards the galactic orbits of stars in the galactic gravitational field, if the ages of the stars are known. For 25 nearby stars of type B8 and B9 with ages derived by Strömgren, Yuan (1969*b*) has shown that their birthplaces can be made to agree with the position of spiral arms only if the gravitational field due to the density wave is taken into account. The significance of this result has been questioned by Contopoulos (1972) and Kalnajs (1973*a*). For 19 classical cepheids with ages derived from the period-age relation, Wielen (1973) has obtained birthplaces mostly in agreement with the predictions of Lin's theory.

E. *Variation of Stellar Distribution with Age.* Stars should be born essentially at the maximum of the density wave. Because the density wave moves in general with a noticeable velocity relative to the material, we expect a systematic spatial separation of young stars according to their age: The spiral arm of the youngest stars should be displaced relative to the arrangement of the slightly older stars. Such an effect seems to be observable in external galaxies (Courtès and Dubout-Crillon 1971; Dixon 1971; Dixon and Ford 1972; Dixon, Ford, and Robertson 1972). The time variation of the places of formation of the stars can lead to a smearing out of the spiral structure among the older objects more rapidly than one would expect from star migration and the velocity dispersion alone (Wielen 1971*b*).

F. *Kinematics of Stars.* The deviations of the mean motion of stars (and gas) from the classical differential rotation because of a density wave has been studied by Rohlfs (1971), Crézé and Mennessier (1973), and Yuan (1974). An explanation of the vertex deviation in the velocity distribution of nearby stars as due to the density wave has been given by Mayor (1970) and Yuan (1971). The increase of the velocity dispersion of stars with age due to an average acceleration by the gravitational field of a density wave has been considered by Barbanis and Woltjer (1967), Marochnik (1969), and Wielen (1974).

G. *Structure and Kinematics of Individual Spiral Arms.* Although the density-wave theory is mainly concerned with the grand design of spiral structure, this theory can also help to understand the detailed structure and kinematics of individual spiral arms on a smaller scale. Roberts (1972) has applied the density-wave theory including shocks with good success to the Perseus arm, which is one of the principal arms of the grand design according to Lin. Humphreys (1972) investigated the Carina spiral feature and found evidence for the presence of a density wave, although the Carina arm may be only a secondary arm according to Lin. Shane (1972) and Simonson and Mader (1973) applied Contopoulos' results on the dispersion rings near the inner Lindblad resonance to the Scutum arm.

Our examples for the application of the density-wave theory are by no means exhaustive. In fact, the density-wave theory is used in many recent papers on galactic research as a convenient frame for a theoretical interpretation of observations. While the density-wave theory is rather successful in giving a unified explanation for many puzzling observations in our Galaxy, the theory is even more suited for an application to external galaxies, where we can better distinguish between the grand design of spiral structure and local irregularities. Many such applications of the density-wave theory to external galaxies have been carried out (e.g., Rogstad 1971; Tully 1972; Warner, Wright, and Baldwin 1973; Guibert 1974; Oort 1974, and other papers already quoted in former sections). Quite probably, the external galaxies will finally provide the most stringent test on the merits as well as on the limits of the density-wave theory.

I would like to thank Professor I. King for his encouragement and for providing his English translation of an earlier review (Wielen 1971*a*). I am also very grateful to Professors C. C. Lin, A. Toomre, G. Contopoulos, A. Kalnajs, W. W. Roberts, F. H. Shu, and C. Yuan for helpful information and comments.

REFERENCES

Ambartsumian, V. A. 1958, in *La Structure et l'Evolution de l'Univers*, R. Stoops, ed. (Brussels: Solvay Institute), p. 241.
Aoki, S. 1974, *Pub. Astr. Soc. Japan* (in press).
Arp, H. 1969*a*, *Astr. and Ap.* **3**, 418.
—— 1969*b*, *Sky and Tel.* **38**, 385.
Barbanis, B. 1970, in *I.A.U. Symposium* No. 38, W. Becker and G. Contopoulos, eds. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 343.
Barbanis, B., and Woltjer, L. 1967, *Ap. J.* **150**, 461.
Becker, W., and Contopoulos, G. (eds.) 1970, *The Spiral Structure of Our Galaxy, I.A.U. Symposium No. 38* (Dordrecht, Holland: D. Reidel Publishing Co.), cited as *I.A.U. Symposium No. 38*.
Bergh, S. van den 1960*a*, *Ap. J.* **131**, 215.
—— 1960*b*, *ibid.* **131**, 558.
Biermann, P., Kippenhahn, R., Tscharnuter, W., and Yorke, H. 1972, *Astr. and Ap.* **19**, 113.
Bok, B. J. 1971, in *Highlights of Astronomy*, Vol. 2, C. de Jager, ed. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 63.
—— 1972, *American Scientist* **60**, 709.
Burbidge, E. M. 1970, *Comments Ap. and Space Sci.* **2**, 25.
Burbidge, G. R. 1969, *Comments Ap. and Space Sci.* **1**, 25.
Burton, W. B. 1971, *Astr. and Ap.* **10**, 76.
—— 1972, *ibid.* **19**, 51.
Burton, W. B., and Shane, W. W. 1970, in *I.A.U. Symposium No. 38*, p. 397.
Contopoulos, G. 1970*a*, in *I.A.U. Symposium No. 38*, p. 303.
—— 1970*b*, *Ap. J.* **160**, 113.
—— 1971, *ibid.* **163**, 181.
—— 1972, "The Dynamics of Spiral Structure", Lecture Notes. (College Park: University of Maryland, Astronomy Program).
—— 1973*a*, in *Dynamical Structure and Evolution of Stellar Systems.* Third Advanced Course of the Swiss Soc. of Astr. and Astrophys., L. Martinet and M. Mayor, eds. (Geneva Observatory), p. 1.
—— 1973*b*, *Ap. J.* **181**, 657.
Courtès, G., and Dubout-Crillon, R. 1971, *Astr. and Ap.* **11**, 468.
Crézé, M., and Mennessier, M. O. 1973, *Astr. and Ap.* **27**, 281.
Dewar, R. L. 1972, *Ap. J.* **174**, 301.
Dixon, M. E. 1971, *Ap. J.* **164**, 411.
Dixon, M. E., and Ford, V. L. 1972, *Ap. J.* **173**, 35.
Dixon, M. E., Ford, V. L., and Robertson, J. W. 1972, *Ap. J.* **174**, 17.
Feldman, S. I., and Lin, C. C. 1973, *Studies in Appl. Math.* **52**, 1.
Freeman, K. C. 1970, in *I.A.U. Symposium No. 38*, p. 351.

Fricke, W. 1952, *A.N.* **280**, 193.
Fujimoto, M. 1968*a*, in *Non-Stable Phenomena in Galaxies, I.A.U. Symposium No. 29,* V. A. Ambartsumian, ed. (Erevan: Publishing House Academy Sci. Armenian SSR), p. 453.
—— 1968*b*, *Ap J.* **152**, 391.
Goldreich, P., and Lynden-Bell, D. 1965, *M.N.R.A.S.* **130**, 125.
Guibert, J. 1974, *Astr. and Ap.* **30**, 353.
Harrison, E. R. 1970, *M.N.R.A.S.* **150**, 443.
Hohl, F. 1970, "Dynamical Evolution of Disk Galaxies," *NASA Technical Report TR R-343.*
—— 1971*a*, *Ap. J.* **168**, 343.
—— 1971*b*, *Ap. and Space Sci.* **14**, 91.
—— 1971*c*, *A.J.* **76**, 202.
—— 1972*a*, *J. Computational Phys.* **9**, 10.
—— 1972*b*, "Computer Experiments on the Structure and Dynamics of Spiral Galaxies", *NASA Technical Note TN D-6630.*
—— 1973, *Ap. J.* **184**, 353.
Hohl, F., and Hockney, R. W. 1969, *J. Computational Phys.* **4**, 306.
Humphreys, R. M. 1972, *Astr. and Ap.* **20**, 29.
Hunter, C. 1963, *M.N.R.A.S.* **126**, 299.
—— 1965, *ibid.* **129**, 321.
—— 1970, in *I.A.U. Symposium No. 38,* p. 326.
—— 1973, *Ap. J.* **181**, 685.
Julian, W. H., and Toomre, A. 1966, *Ap. J.* **146**, 810.
Kalnajs, A. J. 1970, in *I.A.U. Symposium No. 38,* p. 318.
—— 1971, *Ap. J.* **166**, 275.
—— 1972*a*, *ibid.* **175**, 63.
—— 1972*b*, *Ap. Letters* **11**, 41.
—— 1973*a*, *The Observatory* **93**, 39.
—— 1973*b*, *Proc. Astr. Soc. Australia* **2**, 174.
Kato, S. 1971, *Pub. Astr. Soc. Japan* **23**, 467.
—— 1973, *ibid.* **25**, 231.
Kerr, F. J. 1969, *Annual Rev. of Astr. and Astrophysics* **7**, 39.
—— 1970, in *I.A.U. Symposium No. 38,* p. 95.
Kruit, P. C. van der 1973*a*, *Nature Phys. Sci.* **243**, 127.
—— 1973*b*, *Astr. and Ap.* **29**, 263.
—— 1973*c*, *Ap. J.* **186**, 807.
Kruit, P. C. van der, Oort, J. H., and Mathewson, D. S. 1972, *Astr. and Ap.* **21**, 169.
Lin, C. C. 1966, *J. SIAM Appl. Math.* **14**, 876.
—— 1967*a*, *Annual Rev. of Astr. and Astrophysics* **5**, 453.
—— 1967*b*, in *Lectures in Appl. Math.,* Vol. 9 (Relativity Theory and Astrophysics. 2. Galactic Structure), J. Ehlers, ed. (Providence, R. I.: American Mathematical Society), p. 66.
—— 1968, in *Galaxies and the Universe,* L. Woltjer, ed. (New York-London: Columbia University Press), p. 33.
—— 1970*a*, in *I.A.U. Symposium No. 38,* p. 377.
—— 1970*b*, in *Galactic Astronomy,* Vol. 2, H. Y. Chiu and A. Muriel, eds. (New York-London-Paris: Gordon and Breach), p. 1.
—— 1971, in *Highlights of Astronomy,* Vol. 2, C. de Jager, ed. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 88.
Lin, C. C., and Shu, F. H. 1964, *Ap. J.* **140**, 646.
—— 1966, *Proc. Nat. Acad. Sci. USA* **55**, 229.
—— 1967, in *Radio Astronomy and the Galactic System, I.A.U. Symposium No. 31,* H. van Woerden, ed. (London-New York: Academic Press), p. 313.
—— 1971, in *Astrophysics and General Relativity,* Vol. 2, M. Chrétien, S. Deser, and J. Goldstein, eds. (New York-London-Paris: Gordon and Breach), p. 235.
Lin, C. C., Yuan, C., and Shu, F. H. 1969, *Ap. J.* **155**, 721.
Lindblad, B. 1959, *Handbuch der Physik* **53**, 21.
—— 1963, *Stockholm Obs. Ann.* **22**, No. 5.
Lindblad, P. O. 1960, *Stockholm Obs. Ann.* **21**, No. 5.
—— 1962, in *The Distribution and Motion of Interstellar Matter in Galaxies,* L. Woltjer, ed. (New York: W. A. Benjamin Inc.), p. 222.
Lynden-Bell, D. 1970, in *I.A.U. Symposium No. 38,* p. 331.
—— 1973, in *Dynamical Structure and Evolution of Stellar Systems,* Third Advanced Course of the Swiss Soc. of Astr. and Astrophys., L. Martinet and M. Mayor, eds. (Geneva Observatory), p. 91.
Lynden-Bell, D., and Kalnajs, A. J. 1972, *M.N.R.A.S.* **157**, 1.
Lynden-Bell, D., and Ostriker, J. P. 1967, *M.N.R.A.S.* **136**, 293.
Lynds, B. T. 1970, in *I.A.U. Symposium No. 38,* p. 36.
—— 1972, in *External Galaxies and Quasi-Stellar Objects, I.A.U. Symposium No. 44,* D. S. Evans, ed. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 56.
Mark, J. W.-K. 1971, *Proc. Nat. Acad. Sci. USA* **68**, 2095.
Marochnik, L. S. 1969, *Astrofizika* **5**, 487.
Marochnik, L. S., and Suchkov, A. A. 1969*a*, *Ap. and Space Sci.* **4**, 317.
—— 1969*b*, *Astr. Zu.* **46**, 319.
—— 1969*c*, *ibid.* **46**, 524.
Marochnik, L. S., Mishurov, Y. N., and Suchkov, A. A. 1972, *Ap. and Space Sci.* **19**, 285.
Mathewson, D. S., Kruit, P. C. van der, and Brouw, W. N. 1972, *Astr. and Ap.* **17**, 468.
Mayor, M. 1970, *Astr. and Ap.* **6**, 60.
Mezger, P. G. 1970, in *I.A.U. Symposium No. 38,* p. 107.
Miller, R. H. 1971, *Ap. and Space Sci.* **14**, 73.
Miller, R. H., and Prendergast, K. H. 1968, *Ap. J.* **151**, 699.
Miller, R. H., Prendergast, K. H., and Quirk, W. J. 1970, *Ap. J.* **161**, 903.
Miyamoto, M. 1974, *Astr. and Ap.* **30**, 441.
Ng, E. W. 1967, *Ap. J.* **150**, 787.
Nordsieck, K. H. 1973*a*, *Ap. J.* **184**, 719.
—— 1973*b*, *Ap. J.* **184**, 735.
Oort, J. H. 1965, *Stars and Stellar Systems* **5**, 455.
—— 1973*a*, *Mitt. Astron. Ges.* No. 32, 15.
—— 1973*b*, *Commentarii (Pontificia Acad. Sci.)* **2**, No. 55, p. 1.
—— 1974, in *The Formation and Dynamics of Galaxies, I.A.U. Symposium No. 58,* J. R. Shakeshaft, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Ostriker, J. P., and Peebles, P. J. E. 1973, *Ap. J.* **186**, 467.
Perek, L. 1966, *Bull. Astr. Inst. Czechoslovakia* **17**, 333.

Pfleiderer, J. 1963, *Zs. f. Ap.* **58**, 12.
Piddington, J. H. 1973*a*, *M.N.R.A.S.* **162**, 73.
— 1973*b*, *Ap. J.* **179**, 755.
Pikelner, S. B. 1970*a*, *Astr. Zu.* **47**, 752.
— 1970*b*, *Ap. Letters* **7**, 11.
Quirk, W. J. 1971, *Ap. J.* **167**, 7.
Roberts, W. W. 1969, *Ap. J.* **158**, 123.
— 1972, *Ap. J.* **173**, 259.
— 1974, in *Highlights of Astronomy*, Vol. 3, G. Contopoulos, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Roberts, W. W., and Shu, F. H. 1972, *Ap. Letters* **12**, 49.
Roberts, W. W., and Yuan, C. 1970, *Ap. J.* **161**, 877.
Roberts, W. W., Roberts, M. S., and Shu, F. H. 1974, in *The Formation and Dynamics of Galaxies, I.A.U. Symposium No. 58*, J. R. Shakeshaft, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Rogstad, D. H. 1971, *Astr. and Ap.* **13**, 108.
Rohlfs, K. 1971, *Astr. and Ap.* **17**, 246.
Schmidt, M. 1965, *Stars and Stellar Systems* **5**, 513.
Schmidt-Kaler, T., and House, F. 1974, *Astr. and Ap.* (in press).
Shane, W. W. 1972, *Astr. and Ap.* **16**, 118.
Shane, W. W., and Bieger-Smith, G. P. 1966, *Bull. Astr. Inst. Netherlands* **18**, 263.
Shu, F. H. 1969, *Ap. J.* **158**, 505.
— 1970*a*, *ibid.* **160**, 89.
— 1970*b*, *ibid.* **160**, 99.
— 1973*a*, *American Scientist* **61**, 524.
— 1973*b*, in *Interstellar Dust and Related Topics, I.A.U. Symposium No. 52*, J. M. Greenberg and H. C. van de Hulst, eds. (Dordrecht, Holland: D. Reidel Publishing Co.), p. 257.
Shu, F. H., Milione, V., and Roberts, W. W. 1973, *Ap. J.* **183**, 819.
Shu, F. H., Milione, V., Gebel, W., Yuan, C., Goldsmith, D. W., and Roberts, W. W. 1972, *Ap. J.* **173**, 557.
Shu, F. H., Stachnik, R. V., and Yost, J. C. 1971, *Ap. J.* **166**, 465.
Simonson, S. C. 1970, *Astr. and Ap.* **9**, 163.
Simonson, S. C., and Mader, G. L. 1973, *Astr. and Ap.* **27**, 337.
Toomre, A. 1963, *Ap. J.* **138**, 385.
— 1964, *ibid.* **139**, 1217.
— 1969, *ibid.* **158**, 899.
— 1970, in *I.A.U Symposium No. 38*, p. 334.
— 1974, in *Highlights of Astronomy*, Vol. 3, G. Contopoulos, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Toomre, A., and Toomre, J. 1972, *Ap. J.* **178**, 623.
Tosa, M. 1973, *Pub. Astr. Soc. Japan* **25**, 191.
Tully, R. B. 1972, *Bull. A.A.S.* 4, 267.
Vandervoort, P. O. 1970*a*, *Ap. J.* **161**, 67.
— 1970*b*, *ibid.* **161**, 87.
— 1970*c*, *ibid.* **162**, 453.
— 1971, *ibid.* **166**, 37.
— 1973, *ibid.* **180**, 739.
Vaucouleurs, G. de 1959, *Handbuch der Physik* **53**, 275.
— 1963, *Ap. J. Suppl.* **8**, 31.
Vaucouleurs, G. de, and Freeman, K. C. 1972, *Vistas in Astronomy* **14**, 163.
Warner, P. J., Wright, M. C. H., and Baldwin, J. E. 1973, *M.N.R.A.S.* **163**, 163.
Weaver, H. F. 1970, in *I.A.U. Symposium No. 38*, p. 126.
Wielen, R. 1971*a*, *Mitt Astron. Ges.* No. 30, 31.
— 1971*b*, *Astr. and Ap.* **13**, 309.
— 1973, ibid. **25**, 285.
— 1974, in *Highlights of Astronomy*, Vol. 3, G. Contopoulos, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Woltjer, L. 1965, in *Galactic Structure*, A. Blaauw and M. Schmidt, eds. (Chicago: University of Chicago Press), p. 531.
— 1970, in *I.A.U. Symposium No. 38*, p. 439.
Yuan, C. 1969*a*, *Ap. J.* **158**, 871.
— 1969*b*, *ibid* **158**, 889.
— 1971, *A.J.* **76**, 664.
— 1974, in *Highlights of Astronomy*, Vol. 3, G. Contopoulos, ed. (Dordrecht, Holland: D. Reidel Publishing Co.) (in press).
Yuan, C., and Liebovitch, L. S. 1972, *Bull. A.A.S.* **4**, 268.